A PROBABILITY DISTRIBUTION FOR THE NUMBER OF BIRTHS AND ITS APPLICATION

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SUMMARY

The probability model which describes the variation in the number of births to a couple during a specified period of time has been developed under certain assumptions. For the sake of illustration, the probability distribution is applied to the observed data. Its application has also been demonstrated in the evaluation of family planning programme.

In the recent years, a large number of papers which concein with the variation in the number of births to a couple during a specified period of time, have been presented through prbability models by several authors based on varying sets of assumptions. A detailed account of such types of models can be obtained from Brass [1], Dandekar [2], Pathak [4], Singh [6], [7] [8] and Singh *et al.* [11] where the authors have described the literature and added the contributions. These authors have derived the probability distribution for complete conceptions (a conception is called complete if it results in a live birth otherwise, it is called incomplete) to a couple considering one to one correspondence between a conception and a live birth. Singh and Bhattacharya [9] [10]. Sheps and Perrin [5] have considered the distributions incorporating foetal losses also. Though the above mentioned probability distributions have been derived for the number of conceptions to take into account complete as well as incomplete conceptions but due to nonavailability of the data, it may also be applied by taking the two groups with similar characteristics *i.e.*, (a) the infant who dies within a year and (b) the infant who survives atleast one year.

In the above model, it is assumed that the probability of infant who dies within a year is constant throughout the period of observation but several authors like Gulick [3] has reported that the risk of death affects the first born child much more sharply than the second, third, fourth or fifth. Therefore, it may be more realistic to provide the better human fertility phenomenon if we consider the proportion of infant death of the first different from the other order of births. Hence, the purpose of this paper is to highlight on a probability model for the number of births considering two types of deaths and the proportion of the infant who dies within a year is different from the other order of births respectively. The respective probability models are shown in section 2. The application is given in section 3.

SECTION 2

The probability model which describes the variation in the number of births to a female during a specified period of time (O, T) from her marriage, based on the following assumptions:

(1) The female is susceptible to conceive at the beginning of the observational period and has married life throughout the period of observation.

(2*a*) The number of coitions of a couple during any arbitrary time interval (t_1, t_2), $0<t_1<t_2<T$ is a random variable and follows a Poisson distribution with the parameter $m_1 (t_2-t_1)$, $m_1>0$.

(2*b*) The coitions are mutually independent and p_1, the probability that a coition results in a conception is constant.

Under the assumptions (2*a*) and (2*b*), it can easily be seen that the number of conceptions during the time interval (t_1, t_2) follows a Poisson distribution with the parameter $m (t_2-t_1)=m_1p_1 (t_2-t_1)$. It is easy to see that the waiting time of first conception is distributed exponentially. If T_0 denotes the time from marriage to first conception for a female, then

$$P [T_0\leqslant t]=F_0 (t)=(1-e^{-mt}), (t>0)$$

(3) Every conception results in a live birth.

(4) Let θ be the probability that an infant will survive more than a year so that $(1-\theta)$ is the probability of infant who dies within a year.

(5) If there is a conception in a certain unit of time, then there is no another conception during the next h_1 and h_2 units of time where h_1 and h_2 are the rest period (gestation+post partum amenorrhoea) associated with the infant and non-infant deaths respectively.

(6) Let $(1-a_0)$ be the probability that the female is primarily sterile and a_0 is the probability that she is fecund at marriage. Again, let $(1-a_r)$ be the probability that she becomes sterile or chooses to be so till the end of the observational period following the termination of. the r^{th} pregnancy. This may be either due to secondary sterility or due to the adoption of some of the family planning methods of hundred percent effectiveness.

As explained in Singh Bhattacharya [9], the maximum number of births to a female during the time interval (O, T) can not exceed n where $n=[T/h_1]+1$, where $[T/h_1]$ stands for the greatest integer not exceeding to T/h_1.

Under the assumptions 1 to 6, the probability distribution function, denoted by $h_{r+1}(T)$ is given by

$$H_{r+1}(T)=a_0\, a_1 \cdots a_r \left[\sum_{j=0}^{1_r} (J)\theta^j(1-\theta)^{r-j}\left\{ 1-e^{-m(T-jh_2-\overline{r-j}h_1)} \cdot \sum_{s=0}^{r} \frac{\{m(V-jh_2-\overline{r-j}h_1)\}^s}{s!}\right\}\right]$$

where $r=1, 2, 3, 4, \ldots\, n-1$, $1_r=\min. \left[r, \frac{T-rh_1}{h_2-h_1} \right]$ ...(2.1)

The proof of the model (2.1) follows from Singh and Bhattacharya [9].

Further, on taking the probability of first infant that dies within a year is different from other order of births *i.e.* all θ;s may be taken approximately same except θ_1 where θ_1 is the probability that the first infant will survive more than a year. It has also been observed that the incident of secondary sterility prior to age 30 years and hundred percent effective contraceptive practices among the females in rural area are almost negligible. Thus $(1-a_r)$, $r=1,2,\ldots$, may reasonably be assumed zero. Hence, the above probability distribution reduces to the following form :

$$H_{r+1}(T)=a_0\Big[(1-\theta_1) \sum_{j=0}^{1_r-1} \binom{r-1}{j}\theta^j(1-\theta)^{r-j-1} \cdot G+\theta_1 \sum_{j=1}^{1_r} \binom{r-1}{j-1}\, \theta^{j-1}(1-\theta)^{r-j}.\, G \Big]$$

$$\text{where } G=\left[1-e^{-m(T-jh_2-\overline{r-j}h_1)}.\sum_{s=0}^{r}\frac{\{m\,(T-jh_2-\overline{r-j}h_1)\}^s}{s\,!}\right] \quad \ldots(2.2)$$

Hence, the probability of r births to a female during the time interval (O, T) is

$$P_r=H_r\,(T)-H_{r+1}\,(T),\ r=1, 2, 3, \ldots, n-1. \quad \ldots(2.3)$$

$$P_n=H_n\,(T), \text{ where } H_0\,(T)=1,\ H_1\,(T)=a_0\,(1-e^{-mT})$$

SECTION 3

The model involves several parameters. It is difficult to estimate all the parameters h_1, h_2, θ_1, θ, a_0, and m. When any distribution is applied to the observed data, reasonable values of parameters are assumed to be known at the time of estimation. Hence, we assume that h_1, h_2, θ_1, and θ are known. The estimates of the parameters a_0 and n are derived by equating the mean and the relative frequencies of females with zero birth of the observed distribution to the theoretical expressions.

$$\bar{x}=\sum_{r=0}^{n} r.P_r=a_0\,F\,(m) \quad \ldots(3.1)$$

and

$$f_0=1-a_0+a_0e^{-mT} \quad \ldots(3.2)$$

Where $F\,(m)$ denotes the function of m. A detailed account can be obtained from Singh [6].

It is hardly to be emphasized that for the best use and utility of the model, we need data which can be available to us from a special studies designed to meet the purpose. For discussing the applicability of the model, the data on the total number of births during a given period of time could be available to us from a Demographic Survey of Varanasi (Rural) which was conducted in the year (1969-70) under the auspices of the Demographic Research Centre, Banaras Hindu University, Varanasi. A detailed account of Demographic Survey is given in Singh etal [12].

The table 1 shows the observed distribution for the number of births to 494 women during 10 years of period from her marriage. In order to apply the model to the data, we take $h_1=1$ year (9 months for gestation and 3 months for P.P.A.), $h_2=1.25$ years (9 months for gestation and 6 months for P.P.A.), $\theta_1=0.75$, $\theta=0.80$. The estimates of a_0 and m are obtained with the help of the expressions (3.1) and (3.2) as

$$\hat{a}_0=0.9455,\ \hat{m}=0.452$$

TABLE 1

"Distribution of the observed and the expected number of women according to the number of births in ten years from marriage"

No. of birth	*Observed number of women*	*Expected number of women*		
		$\theta=1.00$	$\theta=0.80$	$\theta=0.80,\ \theta_1=0.75$
0	32	32.0	32.0	32.0
1	59	51.7	51.0	50.8
2	127	135.5	131.8	131.4
3	154	162.9	150.1	159.9
4	87	90.4	103.9	93.7
5	32 }	21.5	25.2	26.2
6 and over	3 }			
Total	494	494.0	494.0	494.0
x^2	—	10.65	8.08	5.13
Degree of Freedom	—	3	3	3

The frequencies of the columns 3,4 and 5 in table 1 are calculated assuming no infant death, 20 percent infant deaths throughout the period of observation and 25 percent infant death for the first and 20 percent for remaining order of births respectively. The values of X^2 for the same are 10.65,8.08 and 5.13 respectively. So if agreement between the observed and the expected frequencies is a criterion for the suitability of the model then this suggest that the proposed models under consideration have successfully described the data.

USE OF THE MODEL IN THE EVALUATION OF FAMILY PLANNING PROGRAMME :

In recent years, several developing countries like India have launched family planning programme for fertility control. The

TABLE 2

The probabilities of exactly X births in a period of 20 years under different schemes

No. of birth	*Probabilites under the schemes*						
	A	*B*	*C*	*D*	*E*	*F*	*G*
0	.0001	.0001	.0001	.0001	.0001	.0001	.0001
1	.0019	.0019	.0019	.0019	.0019	.0019	.0019
2	.0127	.0127	.0127	.0127	.0127	.0127	.0127
3	.0492	.1897	.2833	.4237	.5173	.6109	.7511
4	.1240	.2089	.2453	.2593	.2650	.2445	.1833
5	.2107	.2174	.2031	.1624	.1278	.0914	.0414
6	.2459	.1838	.1412	.0838	.0530	.0294	.0080
7	.1958	.1147	.0746	.0337	.0127	.0075	.0012
8	.1087	.0515	.0288	.0100	.0042	.0014	.0001*
9 and over	.0510	.0192	.0090	.0024	.0008	.0002	—
Mean births	5.90	5.01	4.55	4.03	3.76	3.54	3.28
Percentage reduction in births	—	15.1	22.9	31.7	36.3	40.0	44.4

* 8 or more births.

specific devices affect fertility parameters which reduce the fertility. Therefore, it is desirable to develope some techniques to evaluate the short and long term changes in fertility under specified programme. Thus, the probability model which is given in section 2 by the equation (2.1) is useful in this context. The present model can be utilized for measuring the changes in fertility for a group of females during a given time interval since marriage who are using birth control methods of hundred percent effectiveness or trying to limit their family size by sterilization techniques depending on parity.

The table 2 illustrates the percentage reduction in the average number of births and probabilities for different order of births in first 20 years of marriage to couples by assuming the values of $m=0.452, \theta=0.80, h_1=1.00$ year and $h_2=1.25$ years under the following schemes :

(A) : No sterilization and no sterility, *i.e.* $a_r=1, r=0,1,2, \ldots$

(B) : 15 percent of couples are becoming sterile or using hundred percent effective contraceptives at each birth following the birth of the third child and onwards *i.e.* $a_0=a_1=a_2=1$ and $a_r=0.85$ for $r=3,4,5, \ldots$

(C) : 25 percent of couples are either becoming sterile or using hundred percent effective contraceptives at each birth following the third child and onwards *.ie.*, $a_0=a_1=a_2=1$ and $a_r=0.75$ for $r=3,4,5, \ldots$

(D) : After the birth of third child, 40 percent of couples are using hundred percent effective contraceptives or becoming sterile at each birth and onwards ,*ie.*, $a_0=a_1=a_2=1$ and $a_r=0.60$ for $r=3,4,5, \ldots$

Simultaneously 50, 60 and 75 percent couples are either using hundred per cent effective contraceptive devices or they are becoming sterile at each birth after the birth of the third child and onwards *e.i*, $a_0=a_1=a_2=1$ and $a_r=0.50, 0.40$ and 0.25 for $r=3,4,5$, which is shown by the schemes *E, F* and *G* respectively.

Last row of the table 2 shows the percentage reduction in the family size by different schemes. Such as if 15, 25, 40, 50, 60 and 75 percent of couples are either sterilized or using hundred percent effective contraceptive after the birth of the third child then reduction in the average number of births will be 15.1, 22.9, 31.7, 36.3, 40.0 and 44.4 percent respectively during the period of 20 years from marriage. So, from this model, the planners or policy makers may get the better idea about the reduction in the average number of births in different period of times by different schemes and any plan may be made to reduce the birth rate.

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